

## PREFACE

**cartouche:** A tablet for an inscription or for ornament, representing a sheet of paper with the ends rolled up; a drawing or figure of the same, for the title of a map, or the like; a drawn framing of an engraving, etc.

The Physics/Astronomy Building includes a number of special features; prominent among them are the equations and designs on the cartouche around the building. Following the suggestion of Cesar Pelli, the architect of the building, that equations and designs on the cartouche would be a suitable architectural feature, the faculty of the Physics and Astronomy departments were asked to nominate designs for inclusion on the building. I then went through the suggestions and made a selection which was then approved by the Faculty.

This booklet describes what the various cartouche represent. They reflect the state of our knowledge of Physics and Astronomy as of the date of their selection, 1991. Some of the equations are of historical significance, others represent more recent insights and formulations. At least one result is already superceded (see number 4), while others are expected to be valid, at least as a good approximation, as long as physics is studied (for example, Newton's Law of Gravitation, number 32, is known to be only an approximation).

The plan on the preceding page indicates the locations of the cartouche with the numbers indicating the number of the discussion which follows.

David Boulware  
Professor of Physics

- 1) These equations characterize the behavior of a magnetic system near the critical temperature,  $T_c$ , which separates the magnetized and unmagnetized phases. The relations shown describe how the specific heat, the magnetization, and the magnetic susceptibility vary with temperature near the critical point. The same critical exponents,  $(\alpha, \beta, \gamma)$ , characterize the behavior of many different systems undergoing similar order-disorder transitions.

$$\begin{aligned} C(T) &= C_o |T - T_c|^{-\alpha} \\ M(T) &= M_o |T_c - T|^\beta \\ \chi(T) &= \chi_o |T - T_c|^{-\gamma} \\ \alpha + 2\beta + \gamma &= 2 \end{aligned}$$

- 2) This is the second of Maxwell's equations; it describes how changing magnetic fields,  $B$ , generate electrical fields,  $E$ . The relationship describes the principles on which generators and motors work. (The parameter  $c$  is the speed of light.)

$$\nabla \times E + \frac{\partial B}{c \partial t} = 0$$

- 3) This is the first of Maxwell's equations, known as Gauss's Law. It describes how the electric field,  $E$ , is generated by the electrical charge distribution,  $\rho$ .

$$\nabla \cdot E = \rho$$

- 4) This is the current (1991) value of the electron's gyromagnetic ratio; it was measured at the University of Washington by Hans Dehmelt and his collaborators. (The gyromagnetic ratio is a constant which appears in the relation between the spin of a particle and its magnetic moment; the latter determines the strength of the particle's interaction with a magnetic field.)

$$g_e = 2 \quad (1 + .001\,159\,652\,188\,4(43))$$

influence of a force. The function  $f$  is the density of particles in phase space (as a function of position and momentum); this equation tells how the density evolves for a gas of particles moving in an arbitrary potential,  $V$ , but without collisions.

$$\begin{aligned} \partial f / \partial t + p \cdot \nabla_r f / m \\ - \nabla_r V \cdot \nabla_p f = 0 \end{aligned}$$

- 29) This is the so-called action from which the equations for Quantum Electrodynamics, Maxwell's equation and the Dirac equation can be derived. It is written in abstract form with  $F$  representing the electromagnetic field tensor including both electric and magnetic fields.

$$\begin{aligned} W = \int d^4x \{ F^2 / 4e^2 \\ + \bar{\psi} (-i\partial - \not{A} + m) \psi \} \end{aligned}$$

- 30) This is the basic relation of statistical mechanics which relates the partition function,  $Z$ , to the Hamiltonian,  $H$ , and the free energy,  $F$ , for a system in thermodynamic equilibrium at a temperature  $T$ ; the constant  $k$  is Boltzmann's constant which relates temperature to energy.

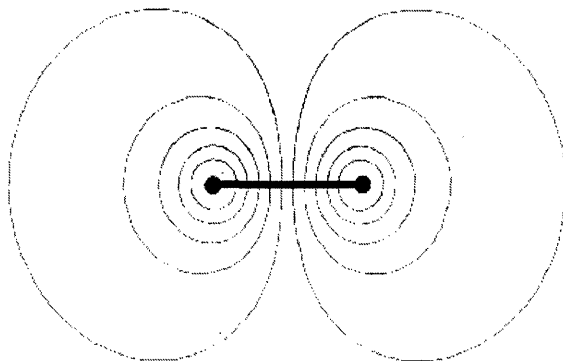
$$Z = \text{Tr} e^{-H/kT} = e^{-F/kT}$$

- 31) This is the relativistic form of the two Maxwell's equations which tell how charges and currents are related to the electric and magnetic fields. (See numbers 3 and 6.) The field  $F$  includes both the electric and the magnetic fields and the current  $j$  includes both the charge density and the electric current density.

$$\partial_\nu F^{\mu\nu} = j^\mu$$

- 32) This is Newton's law of gravitation which describes the gravitational force,  $F$ , between two masses,  $M_1$  and  $M_2$ . The constant  $G$  is Newton's

24) This represents the magnetic field of a current loop.



25) The equation is known as the Dirac equation. The solutions to this equation describe the quantum mechanical behavior of electrons and positrons in an arbitrary electromagnetic field which is described by  $A$ . The equation is the special relativistic analogue of Schrödinger's Equation, (9).

$$(\not{p} - e\mathcal{A} + m)\psi = 0$$

26) When a quantum mechanical system is described by a density matrix, this equation relates the entropy,  $S$ , to the density matrix,  $\rho$ .

$$S = -\text{Tr } \rho \ln \rho$$

27) This is the quantum of magnetic flux which appears in superconductivity. The magnetic field must be such that the magnetic flux,  $\Phi_0$ , through a surface is exactly an integer multiple of this flux. The constants  $h$ ,  $c$ , and  $e$  are respectively Planck's constant, the speed of light, and the electron charge.

$$\Phi_0 = \frac{hc}{e}$$

28) This is the Boltzmann equation which describes the statistical mechanical properties of a gas of otherwise free particles moving under the

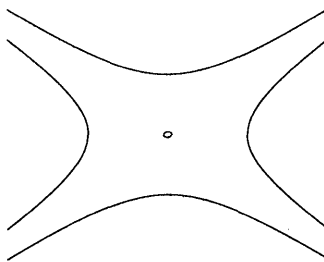
attraction. The equation relates the curvature of space-time to the matter distribution.

$$G^{\mu\nu} = 8\pi GT^{\mu\nu}$$

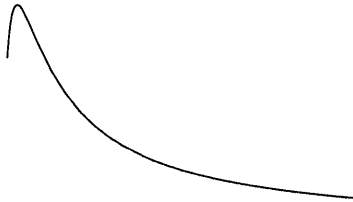
- 18) This equation describes the relation between potential difference,  $\Delta\phi$ , and the frequency,  $\nu$ , in the Josephson effect. The constants  $h$  and  $e$  are the Planck's constant (times  $2\pi$ ) and the electron charge respectively.

$$\Delta\phi = \frac{h}{e}\nu$$

- 19) This is the same as number 2.  
20) This is the same as number 3.  
21) This is the same as number 5.  
22) This is the same as number 6.  
23) This is a graphic representation of the Penning trap in which Hans Dehmelt and his collaborators trapped a single electron and measured its properties to unprecedented accuracy.



- 14) This represents the data from the COBE satellite showing that the cosmic 3°K radiation is, to extremely high accuracy, blackbody radiation. This radiation is the remnant of the ‘big bang’.



- 15) This is the relativistic relation between energy  $E$  and momentum  $p$  of a particle; it is most commonly quoted as  $E = mc^2$ , where  $m$  is the mass of the particle and  $c$  is the speed of light.

$$E^2 = p^2c^2 + m^2c^4$$

- 16) This is a fundamental relation of quantum mechanics. The quantities  $q$  and  $p$  are ‘operators’ representing position and momentum and the fact that they give different results when multiplied in the two orders is the statement that they are quantum mechanical rather than ordinary numbers. The quantity  $\hbar$  is Planck’s constant which sets the fundamental scale of quantum mechanics.

$$qp - pq = i\hbar$$

- 17) This is the fully covariant form of the equations describing Einstein’s General Theory of Relativity. The left side of the equation,  $G^{\mu\nu}$  is the so-called Einstein tensor which describes the curvature of space-time. The right side of the equation is the stress-energy tensor,  $T^{\mu\nu}$ , which describes how matter is distributed in space-time. The constant  $G$  is Newton’s constant which determines the strength of gravitational

- 12) This is Heisenberg's Uncertainty Principle. It states that the uncertainty in the position,  $\Delta q$ , times the uncertainty in the momentum,  $\Delta p$  is equal to Planck's constant,  $\hbar$ , divided by 2. Planck's constant sets the scales at which quantum mechanics becomes important. This is a fundamental statement of our inability in principle to specify both the position and the momentum (velocity) of any body. For macroscopic bodies, the uncertainty is too small to be significant, but inside the atom it completely changes the behavior expected (and found).

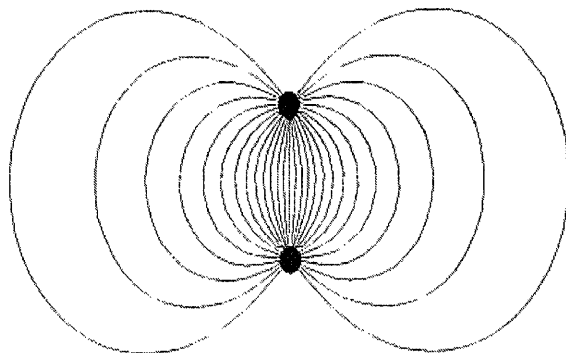
$$\Delta q \Delta p \geq \hbar/2$$

- 13) This is the Hertzsprung-Russell diagram which shows the relation between the color of stars and their absolute magnitude. It helps us to understand the evolution of stars.





- 8) This is a representation of the electric field of an electric dipole.



- 9) This is Schrödinger's equation, the basic equation describing a quantum mechanical non-relativistic particle of mass  $m$  moving in a potential  $V$ . The square of the wave function  $\psi$  gives the probability of finding the particle at a particular point and  $\hbar$  is Planck's constant which sets the scales at which quantum mechanics becomes important. The symbols  $\partial/\partial t$  and  $\nabla$  respectively represent derivatives with respect to time and position.

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2 \nabla^2 \psi}{2m} + V\psi$$

- 10) This one of the fundamental relations of thermodynamics; it gives the relation between the change in the entropy,  $\Delta S$ , and the heat,  $\Delta Q$ , transferred to the system at temperature  $T$ . The constant  $k$  is Boltzmann's constant which relates the temperature scale to energy.

$$\Delta S = \frac{\Delta Q}{kT}$$

- 11) This is Newton's second law which is one of the three foundation principles of classical mechanics, relating the acceleration,  $a$ , of a body to the force,  $F$ , acting on the body and the mass,  $m$ , of the body.

$$F = ma$$

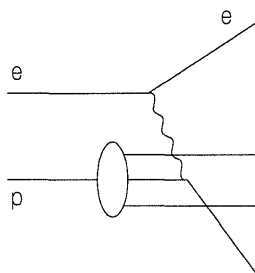
- 5) This equation is the third of Maxwell's equations. It states that there are no magnetic charges and, in combination with the fourth equation, that magnetic fields,  $B$ , are only produced by moving electric charges and time-varying electric fields.

$$\nabla \cdot B = 0$$

- 6) This is the fourth of Maxwell's equations; it describes how magnetic fields,  $B$ , are related to the electric currents,  $j$ , which produce them and how they are related to the time varying electric field,  $E$ . The latter relationship is responsible for the existence of light as an electromagnetic wave and the constant  $c$  is the speed of light.

$$\nabla \times B - \frac{\partial E}{c \partial t} = \frac{j}{c}$$

- 7) This is a 'Feynman diagram' representing a so-called deep inelastic scattering of an high energy electron off a proton. The wavy line represents the exchanged photon and the three lines represent the quarks from the proton. The upper line represents the incoming and outgoing electron. This process is central to our understanding that nucleons (protons and neutrons) are composed of quarks.



constant which describes the strength of the gravitational interaction and  $r$  is the distance between the masses.

$$F = G \frac{M_1 M_2}{r^2}$$

- 33) This equation gives the Chandrasekar limit, the maximum mass that a star can have when it has collapsed so far that the quantum mechanical pressure generated because the electrons cannot be in the same state prevents the star from collapsing further. It is consistent with the observed masses of white dwarfs. The equation describes a star consisting of nuclei with  $Z$  protons and  $A - Z$  neutrons; the mass of a proton is  $m_p$  and the constants  $h$ ,  $c$ , and  $G$  are respectively Planck's constant (see 12), the speed of light, and Newton's constant (see 32).

$$M_{\text{Ch}} \sim \left(\frac{Z}{A}\right)^2 \left(\frac{hc}{Gm_p^2}\right)^{\frac{3}{2}} m_p$$